2nd IST Austria Summer School in Analysis and PDEs

Organizers: Laszlo Erdős, Julian Fischer, Jan Maas, Robert Seiringer

Date: July 25–29, 2022

Lecture series:

- Felix Otto, MPI for Mathematics in the Sciences
  Partial Differential Equations and Randomness
- Wilhelm Schlag, Yale University
  On Long-Term Dynamics of Nonlinear Dispersive Evolution PDEs
- László Székelyhidi, University of Leipzig
  The Onsager Conjecture

Invited talks:

- Peter Bella, Univ. Dortmund
  Regularity Theory of Degenerate Elliptic Equations
- Jonas Lührmann, Texas A&M University
  On asymptotic stability of solitons in classical scalar field theories on the line
- Stefano Modena, Univ. Darmstadt
  On the uniqueness and non-uniqueness of solutions to the linear transport equation

In addition, there will be about 5 lightning talks (ca. 10 minutes per talk) by PhD students on Thursday.

Schedule

Monday

9:30 - 9:35	Opening
9:35 - 10:50	Lecture 1.1 – Wilhelm Schlag
10:50 - 11:30	Coffee Break
11:30 - 12:45	Lecture 2.1 - Laszlo Szekelyhidi
12:45 - 14:30	Lunch
14:30 - 15:15	Lecture 1.2 - Wilhelm Schlag
15:15 - 15:45	Coffee Break
15:45 - 16:45	Lecture 2.2 - Laszlo Szekelyhidi

Tuesday

9:30 - 10:45	Lecture 2.3 - Laszlo Szekelyhidi
10:45 - 11:30	Coffee Break
11:30 - 12:45	Lecture 1.3 - Wilhelm Schlag
12:45 - 14:30	Lunch
14:30 - 15:30	Lecture 2.4 - Laszlo Szekelyhidi
15:30 - 16:00	Coffee Break
16:00 - 16:45	Invited Talk - Jonas Lührmann

Wednesday

9:30 - 10:45	Lecture 3.1 - Felix Otto
10:45 - 11:30	Coffee Break
11:30 - 12:45	Lecture 1.4 - Wilhelm Schlag
12:45 - 14:15	Lunch
14:30 -	Excursion and Dinner at Redlinger Hütte

Thursday

9:30 - 11:00	Lecture 3.2 - Felix Otto
11:00 - 11:45	Coffee Break
11:45 - 12:45	Invited Talk - Stefano Modena
12:45 - 14:30	Lunch
14:30 - 15:30	Lightning talks by PhD students
15:30 - 16:00	Coffee Break
16:00 - 16:45	Lecture 3.3 - Felix Otto
17:30	Guided tour at the Stift and Dinner

Friday

9:30 - 10:30	Invited Talk - Peter Bella
10:30 - 11:15	Coffee Break
11:15 - 12:45	Lecture 3.4 - Felix Otto
12:45 - 12:50	Closing
12:45	Lunch

Lightning talks by PhD students

On Thursday, there will be 5 lightning talks (10 minutes) by PhD students.

14:30 *Daniel Boutros, Univ. Cambridge*
On Energy Conservation for the Hydrostatic Euler Equations: An Onsager Conjecture

14:42 *Charlotte Dietze, Univ. Munich*
Dispersive Estimates for Nonlinear Schrödinger Equations with External Potentials

14:54 *Alice Marveggio, ISTA*
Quantitative convergence of the vectorial Allen-Cahn equation towards multiphase mean curvature flow

15:06 *Kihoon Seong, KAIST*
Transport properties of Gibbs and Gaussian measures under the flow of Hamiltonian PDEs

15:18 *Nicolas Camps, Univ. Paris-Saclay*
NLS on the sphere with random initial data

Abstracts

PDEs and randomness

Felix Otto, MPI for Mathematics in the Sciences

Consider the quasi-linear parabolic equation $\partial_t u - a(u)\partial_x^2 u = \xi$ driven by white noise ξ. This right-hand side is so irregular that the product $a(u)\partial_x^2 u$ requires what is called a renormalization. By this one means that as a regularization of the noise, e. g. by convolution on scale ϵ, vanishes, the PDE has to me modified – as little as possible – by a divergent counter term in form of $\partial_t u - a(u)\partial_x^2 u + h_\epsilon(u) = \xi_\epsilon$ in order for the solution manifold to stay controlled, while sharing as many symmetries as possible of the original equation.

Hairer's regularity structures are designed to tackle such a task. In this mini-course, I shall give an efficient but self-contained introduction into regularity structures with an eye on the above class of problems. I shall do so in a way that is more analytical and less combinatorial than the standard approach, hoping that this is more appealing to PDE analysts.

More specifically, I shall focus on how to choose the counter-term h_ϵ as a function of the nonlinearity a, and on how to stochastically estimate the solution manifold, as encoded term-by-term by what is called the model, uniformly in ϵ. For the experts: We provide an alternative approach to the results contained in Chandra-Hairer '16, which themselves are an extension of substantial work in mathematical physics. Some of our methods are imported from quantitative stochastic homogenization, like the usage of Malliavin calculus and the spectral gap inequality, annealed estimates, and tackling large alongside small scales.

To get an idea, I recommend to have a look at the arXiv-paper "A diagram-free approach to the stochastic estimates in regularity structures", with Linares, Tempelmayr, and Tsatsoulis.

On Long-Term Dynamics of Nonlinear Dispersive Evolution PDEs

Wilhelm Schlag, Yale University

This mini course will present some aspects of Hamiltonian evolution equations such as the wave map system. We will briefly review basics of Hamiltonian dynamics, and then move on to linear and nonlinear wave equations. Harmonic maps arise as stationary solutions of wave maps, and we will review harmonic maps from the 2-sphere to itself. The soliton resolution theorem by Jendrej, Lawrie for equivariant wave maps from 2021 will be presented. Throughout, the emphasis lies on the asymptotic description of the solutions to nonlinear wave equations. We will therefore discuss basic notions from dynamical systems related to the long-term analysis of trajectories, and the stability of equilibria. The course will conclude with a discussion of the 9-set theorem of Nakanishi and the speaker form 2010.

The Onsager conjecture

László Székelyhidi, University of Leipzig

tba

Regularity Theory of Degenerate Elliptic Equations

Peter Bella, University of Dortmund

I will discuss local regularity properties of solutions of linear non-uniformly elliptic equations with non-constant coefficients. Assuming certain integrability conditions on the ellipticity of the coefficient field, we obtain local boundedness of weak solutions and corresponding Harnack inequality. The assumed integrability assumptions are sharp and improve upon classical work [Trudinger, ARMA 1971]. Depending on the time I might discuss application to quenched invariance principle for random walks among random degenerate conductances, regularity of minimizers for scalar integral functionals with differential (p,q)-growth, and Liouville's Theorem.

On asymptotic stability of solitons in classical scalar field theories on the line

Jonas Luehrmann, Texas A&M University

In this talk we consider classical relativistic scalar field theories in one space dimension. Prime examples include the sine-Gordon model and the ϕ^4 model. Their field equations admit topological soliton solutions called kinks, which are expected to form the building blocks for the long-time dynamics of typical solutions.
We begin with a general introduction to classical scalar field theories on the line. Then we present a perturbative proof of the asymptotic stability of the sine-Gordon kink under odd perturbations

(joint work with W. Schlag). In this context we discuss several fundamental techniques to study the decay and the asymptotics of dispersive waves in the presence of strong nonlinear interactions. We conclude with a discussion of some open problems.

On the uniqueness and non-uniqueness of solutions to the linear transport equation

Stefano Modena, University of Darmstadt

One of the main questions in the theory of the linear transport equation is whether uniqueness of weak solutions to the Cauchy problem holds in the case the given vector field is not smooth. In the talk I will provide an overview on some results obtained in the last few years, showing that even for incompressible, Sobolev (thus quite "well-behaved") vector fields, uniqueness of solutions can drastically fail. This result can be seen as a counterpart to DiPerna and Lions' well-posedness theorem (joint with G. Sattig and L. Székelyhidi).